The Bedmaker Effect December 2010

During recent cold weather I've noticed that there's a glitch at 07h40 every morning, visible here http://www.trin.cam.ac.uk/clock/data.html?from=09/12/2010&xmin=7&xmax=9&channel=amp&channel2=going&skip=5&scale=auto as a sudden steady rate of rising in the pendulum swing amplitude (about +0.015 mrad per minute) and a sudden downward jump in the Going (about -500 ms/day) . What could be causing this?

Options include:

1. an increase in driving torque, for instance due to the release of the strike silencing mechanism which happens at about this time. The problem with this theory is that an increase in driving torque causes a decrease in swing amplitude – rather counter-intuitive, but it is because of pallet friction in the gravity escapement. This can be seen by applying the maintaining weight, as was done here http://www.trin.cam.ac.uk/clock/data.html?from=08/12/2010&xmin=16&xmax=21&channel=amp&channel2=going&skip=20&scale=auto , over a period of 18 minutes. It seems that the maintaining weight reduces pendulum amplitude at a rate of 0.01mrad/min. The other interesting thing to note is that the going is not affected by the increase in torque. So this option can be rejected.

2. sun shining through the window – well, sunrise is at about the right time but the sun doesn't actually get around to shining through the window until about 08h30 because of the shadow cast by the south-east turret of the clock tower. So this option can also be rejected.

3. The bedmaker opens the door of the room below causing some effect (to be determined). This has merit as an explanation because the timing is perfect. She does indeed open the door – and leaves it open – between 07h40 and 07h50 every weekday, which explains why the effect is not observed on the weekends. Moreover, Dr Hopkinson does not leave the door open when he enters so the effect is not observed at other times of the day.

If we go for Option 3, then what is happening? Here is my guess: On a cold night cold air drops down from the unheated clock room into the pendulum chamber – we know this is true because cold air can be felt blowing out of the tiny gaps in the case in the room below. Now, when the bedmaker opens the door around 07h40 in the morning the central heating has been on for an hour or so, but the room is not itself warm enough to force warm air backwards up the pendulum chamber. The key thing is that she opens the room to the whole height of the warm staircase (the building is 3 storeys high). There is now a strong buoyancy pressure to push warm air back up through the tiny gaps so the air in the pendulum chamber gets warmer. The question to ask now is what effect does this have on the pendulum swing amplitude and what effect does it have on the Going? Are these effects calculable? If so, to they agree with observations?

Swing Amplitude: The only way that a change in air temperature in the pendulum chamber can affect swing amplitude is by altering the drag. We can work out (see Appendix below) that a +1C temperature rise will cause a +0.05mrad step change in pendulum amplitude. So there is every possibility that this is what is happening.

Going: There are several ways in which warm air can cause a change in Going: A/ change in buoyancy, B/ change in drag, C/ change in added mass, D/ change in length of pendulum. It turns out that the effects of A, B and C all lump together and these work out at about +30ms/day per degreeC. But note the + sign. This means that the effect is in the wrong direction. Also, the magnitude is far too small – we'd need a 15C temperature rise to make the 500ms/day change that we observe. So it cannot be this effect. That leaves D. But we have a temperature compensated pendulum, so surely there can be no effect? Well, the temperature compensation assumes that all the components of the pendulum are at the same temperature. If the air temperature rises suddenly then the outer tube will warm up before the inner parts can catch up. Let's suppose that the steel outer tube expands all on its own. It turns out that the going will change at –520ms/day per degreeC – which is perfect! Not only is the sign correct, but all we need for our observed –500ms/day is a temperature rise of the steel equal to +1C. This sounds perfectly possible.

What about the rate of change of amplitude? Well, this is tricky to compute, but important if the story so far is to be believable. If we consider air drag on the pendulum then pendulum theory shows that drag dissipates about 0.32mJ of energy per swing cycle. This fixes the amplitude at a certain value. If suddenly the air density were to increase then with a fixed amount of available power the pendulum amplitude must decrease. The rate of decrease depends on the stored kinetic energy in the pendulum bob. Working this all out is interesting, the conclusion being that a +1C rise in air temperature will cause a rate of amplitude gain of +0.0005 mrad/minute. We'd need a +30C to achieve the observed +0.015 mrad per minute. So we're not quite there in thinking that we have the right answer. What I feel sure about is that there is a mechanism somewhere that links a rising air temperature to an increase in swing amplitude.

CONCLUSION

So it is supposed that the opening of the door causes warm air to forced into the pendulum chamber causing amplitude to increase due to reduced drag (or some other mechanism) and going to decrease because the outer tube of the pendulum warms up - it takes a while for the temperature compensation to reach equilibrium. A good test to do on a cold morning will be to leave the door open for long enough for an equilibrium to be reached.

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APPENDIX

A1. Amplitude vs Air Temperature

We know from pendulum theory (see http://www.trin.cam.ac.uk/clock/theory/pendulum.pdf) that the amplitude changes with air density as dA/d(rho) = -A/(3*rho) where A is swing amplitude in radians and rho is air density in kg/m^3. We also know from the gas law that p=rho*R*T where p is pressure, R is the gas constant and T is air temperature. So d(rho)/dT = -rho/T. So dA/dT = A/(3*T). This means that a +1C temperature rise will cause a +0.05 step change in pendulum amplitude.

A2. Going vs Air Temperature

A2(a) the effect of buoyancy, drag and added mass. We know from the effect of air pressure that d(Going)/dp = -8 ms/day per mbar. This is what we designed the barometric compensator for, see http://www.trin.cam.ac.uk/clock/barocompensator.html . Using the Gas law again we get that d(Going)/dT = - p/T * d(Going)/dp and for p=1000mbar and T=300K this gives d(Going)/dT = + 30 ms/day per degreeC

A2(b) the effect of thermal expansion. If the coefficient of thermal expansion for the pendulum is alpha, then d(Going)/dT = -alpha/2, from pendulum theory. With alpha = 12e-6 / degC for steel then we get d(Going)/dT = -12e-6 * 24*60*60 * 1000 = -520 ms/day per degreeC.

A3. Rate of change of Amplitude with a step change in air temperature

Pendulum theory gives that the energy lost to air drag per swing cycle (3 seconds) is E = 4/3*Cd*S*rho*L^2*A^3*(2*pi/To)^2 where Cd is the drag coefficient (let's use Cd=0.5), S is the frontal area of the pendulum bob (let's use S=0.1m^2), L is the pendulum length (L=2m) A is the swing amplitude (A=0.05rad) and To is the complete period (To=3s). Suppose now an increase in air density d(rho), then there will be a fall in the amount of energy available to swing the pendulum, dE/d(rho) = -E/rho. The peak kinetic energy in the swinging bob is KE=0.5*m*(L*(2*pi/To)*A)^2 and the change in KE for a change in amplitude is d(KE)/dA = m*(L*2*pi/To)^2*A . If we match the change in available energy dE with the change in kinetic energy d(KE) then d(KE)/dA = dE/d(rho) and with d(rho)/dT = -rho/T as before in A1, we then get dA/dT = E/(T*m*(L*2*pi/To)^2*A) and with E = 4/3*Cd*S*rho*L^3*A^3*(2*pi/To)^2 we get dA/dT = 4/3*Cd*S*L*rho*A^2/(T*m*Te)

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