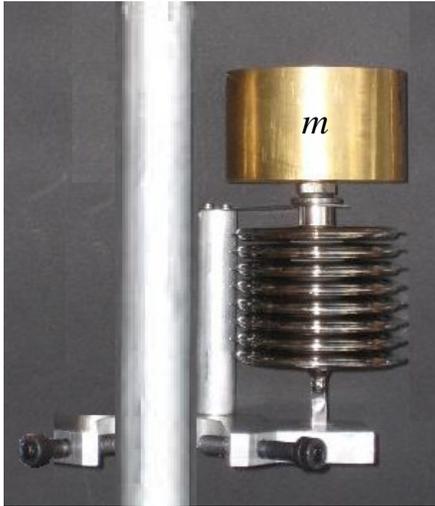


Variation of air density affects the period of a pendulum due to the combined effects of buoyancy, drag and added mass. Air density changes with temperature, barometric pressure and humidity. The effects of temperature change are fully compensated by means of a bi-metal pendulum. The effects of humidity variation are small. This leaves barometric pressure whose effect on “going” is typically around -10ms/day for every millibar rise in pressure. A barometric compensator attached to the pendulum can be designed to negate this effect.



The picture shows a barometric compensator attached to a pendulum shaft. A mass  $m$  is suspended on an aneroid stack whose length contracts by  $\alpha$  mm per mbar rise in atmospheric pressure. The question now is to determine the value of the compensator mass for a given value of  $\alpha$  and also to determine the position on the pendulum rod for the compensator to be attached for optimum performance. First we examine the effect of adding a mass  $m$  at distance  $x$  from the pivot of a pendulum of mass  $M$  and of length  $L$ . The combined moment of inertia of the two masses is  $(ML^2 + mx^2)$  and the restoring torque due to gravity is  $(ML + mx)\sin\theta$  from which the small-amplitude period of the pendulum  $T$  is given by

$$T = 2\pi \sqrt{\frac{ML^2 + mx^2}{MgL + mgx}} \tag{1}$$

and for  $x=0$  or  $m=0$  this gives the standard formula for the unadjusted period  $T_o = 2\pi \sqrt{\frac{L}{g}}$  (2)

Now for convenience define  $z = x/L$  and  $\mu = m/M$  so that

$$T = T_o \sqrt{\frac{1 + \mu z^2}{1 + \mu z}} \approx T_o \left( 1 + \frac{1}{2}(\mu z^2 - \mu z) \right) \tag{3} \approx (4)$$

The binomial theorem has been used here knowing that  $\mu$  is small (the binomial theorem states that  $(1+\epsilon)^n \approx 1+n\epsilon$  for small  $\epsilon$ ). The “going”  $G$  is defined as the deviation of  $T$  from  $T_o$  as a proportion of  $T_o$  so that

$$G = -\frac{T - T_o}{T_o} \approx -\frac{1}{2}(\mu z^2 - \mu z) = \frac{\mu}{2}(1 - z)z \tag{5} \approx (6) = (7)$$

and the minus sign comes about because positive “going” means a *reduction* in period. It helps to plot a graph of Going  $G$  vs Position  $z$ . The value of Going is zero at  $z=0$  and at  $z=1$  (ie points A and B on the graph) and it is maximum at the centre C of the pendulum rod. This explains why it makes sense to regulate the pendulum by “adding pennies” at the *centre* of the rod. Note also that at C the effect of moving a penny up or down is negligible because the curve is at its maximum. The biggest slope of the curve is at points A and B. It turns out that a barometric compensator at B has the wrong sign (makes the problem worse) so we put it at close as possible to the pivot (this is completely counterintuitive). We now need to know the gradient of the curve at A. To do this we take the derivitave  $\frac{dG}{dz} = \mu(\frac{1}{2} - z)$  (8) and at A put

$z=0$  and  $dz = dx/L$  so that  $\frac{dG}{dx} = \frac{\mu}{2L}$  (9). The aneroid stack has sensitivity  $\alpha = \frac{dx}{dp}$  (10) so the

final result is that a barometric compensator has effectiveness

$$\frac{dG}{dp} = \frac{\alpha m}{2LM} \text{ measured in ms/day per millibar} \tag{11}$$

For the Trinity Clock the mass of the bob is estimated at  $M = 80\text{kg}$  and the length of the pendulum  $L = 2.25\text{m}$  (check that this gives a period of 1.5 seconds). We have fitted two compensators side by side, each of mass  $m = 0.75\text{kg}$  and the aneroid stacks have sensitivity estimated at  $\alpha = 0.022\text{mm per mbar}$  (ie a change in length of 1.3mm between “Stormy” and “Very dry”, being about 60mbar on the barometer). This gives a compensation effect of  $\frac{dG}{dp} = \frac{0.022 \times 1.5}{2 \times 2.2 \times 80} \times 24 \times 3600 = 8.1 \text{ ms/day per mbar}$  (12). This is just what we need.

