THE LEIBNIZ-CLARKE CORRESPONDENCE

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As Newton realized, his absolute space was a 'conspiracy of nature' in the sense that his laws dictated that nobody could discover who, among all possible observers (in various states of motion relative to one another), was at rest in absolute space. So absolute space was an unverifiable element of his theory.

So when relativity, with its denial of absolute space, flourished, methodologists amongst the logical empiricists, e.g. Reichenbach, were happy to criticize Newton as a poor methodologist (and to praise his critics on this score, e.g. Leibniz, Berkeley and Mach).

To understand why this criticism is grossly unjust, we first need ...

Part A: Philosophy of Space and Time: An Overview

(AI) Prelude: 'Hymn to Einstein'

(0: Introduction) Einstein's theories of relativity revolutionised not only physics, but also philosophy. In physics, they gave rise to a stream of technical methods and results, both theoretical and observational. Indeed, the stream is now a broad river, which shows now sign of diminishing.

But in philosophy also, Einstein's theories have had an immense influence. Contemporary philosophy of space and time is to a great extent devoted to exploring the philosophical implications of relativity. In particular, it explores how relativity bears on traditional empiricist, especially positivist, worries about the ontological status of space and time, and of geometry. (Such worries of course predate the logical positivists; they date from at least Leibniz, Berkeley and Mach.) Accordingly, I will outline the tradition of philosophy of space and time arising from Einstein's theories.

I shall pick out four broad ideas. The first two come from special relativity (and are retained in general relativity): they are the idea of spacetime and the abandonment of absolute simultaneity. The other two come from general relativity.

(1: Spacetime) So first, the idea of spacetime. Instead of space as a three-dimensional infinite void (points labelled if you like by triples of real numbers), enduring through time (again labelled with real numbers), there is a four-dimensional spacetime: of which space and time are just aspects. Figure 1: a light cone.

With hindsight, we can now see that the pre-Einsteinian conceptions of space and time can also be presented as conceptions of spacetime, and that it is illuminating to do so: this was one of the great contributions of the French geometer Cartan in the 1920s. (Thus people speak of 'Newtonian spacetime', 'Galiliean spacetime' etc.) But there is a profound reason why the idea of spacetime first arose in special relativity, and that relates to our second idea.

(2: Relativity of Simultaneity) The second idea is special relativity's abandonment of the notion of absolute simultaneity, the notion that two mutually distant events are either simultaneous or not, irrespective of anything else. In 1905, Einstein saw, not only that whether two events are absolutely simultaneous could not be established experimentally (an insight shared by some contemporaries), but also that the two great theories of physics, mechanics and electromagnetic theory, could survive without absolute simultaneity.

This truly profound insight took some time to sink in; but over the next few years, its correctness and depth became clearer and clearer. It emerged that mechanics and electromagnetic theory only need a single notion of length, or interval -- but one that is defined on the four-dimensional spacetime, and that itself defines as derived concepts the traditional ideas of spatial length and temporal interval. Figure 2: Minkowski interval.

These latter are derived both in the sense of being mathematically fixed or determined by the spatiotemporal interval, and in the sense of being specific or 'low-level' -- for they really make sense only once one has chosen a coordinate system (reference-frame). Figure 3 shows two choices of simultaneity slices and two choices of a congruence of spatial points.

On the other hand, in the 'pre-Einsteinian spacetimes' defined by Cartan and his followers, the presence of absolute simultaneity means that the spacetime has two metrical structures, one for space and one for time, neither of which determines the other; and both of which make perfectly good sense without any choice of coordinate system. Figure 4: a spacetime with absolute, ie coordinate-independent simultaneity slices and spatial points.

It is in this sense that in special relativity, though not in these other spacetimes, space and time become just *aspects* of spacetime. In the *locus classicus* of this view, Minkowski (1908) writes: “Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality”. Hence people now speak of ‘Minkowski spacetime’, ‘Minkowski interval/metric’ etc. (Remarkably, the mathematics, though not the interpretation, was already in a 1905 paper of Poincaré’s!)

Einstein’s insight that the spacetime of special relativity — Minkowski spacetime — is sufficient for physics is quite astonishing when one considers that it has remained sufficient, not just for other fields of classical physics such as thermodynamics, but also for all the myriad developments of quantum theory, including the ‘strong’ and ‘weak’ forces, unknown to classical physics, that bind atoms together. Sufficient, indeed, for the physics of all known forces, apart from gravity: for which Einstein himself gave us another theory -- his general relativity.

A philosophical moral: Physical Geometry is not a Priori:--
To measure SR's spatiotemporal length we use just the measurement procedures that we intuitively think of as manifesting Newtonian (or neoNewtonian) spacetime structure! So SR implies that our procedures are not logically bound to deliver the space and time structure that we grow up to believe intuitively that the world has.

There is a subtlety here. SR revises our idea of the way space and time are put together: each now seems an aspect of spacetime. But in a sense, SR does not revise our idea of space and time, considered in themselves. That is, time remains ordered like the reals; and space remains Euclidean. GR will be more radical!

(3: Desiderata for General Relativity)
For general relativity, we pick out two broad ideas:

(i) the idea that spacetime structure is influenced by the distribution of matter within it (that ‘spacetime is dynamical’); and

(ii) the idea that gravity is not a force but rather an aspect of spacetime geometry.

As we shall see, both ideas exploit the idea of a curved, non-Euclidean geometry.

It is helpful to approach these two ideas by considering Einstein’s objections to another, apparently natural, approach to finding a theory of gravity that incorporates the insights of special relativity.

This approach conceives gravity as a field on Minkowski spacetime. (This is analogous to Newtonian theory, where gravity is a field on Newtonian spacetime: the field at a point gives the net gravitational force felt by a body of unit mass, placed at the point.). One then seeks equations according to which gravitational influences propagate, not instantaneously as in Newtonian theory, but at some finite speed. (Presumably, one will choose the speed of light, which is already coded into Minkowski spacetime: this choice thus avoids introducing another fundamental speed into physics; and being a high speed, it promises to help explain the empirical success of Newton’s theory.)

In the years after 1905, this approach was tried (Nordstrom 1910). But not by Einstein. For this approach had three central features in common with Newton’s theory: features which Einstein, influenced by Mach, disliked and wanted to abandon.

First, both Newtonian theory and this approach have no explanation of a curious coincidence: that the inertial mass of a body, which underlies its resistance to acceleration under *any* force, is equal to its gravitational mass, i.e. the 'charge' with which it gravitationally attracts and is attracted to other bodies.

There is no *a priori* reason why this equality should hold. Conceptually, a body's inertial mass has no closer relation to its gravitational mass than to its electric charge (by which it electrostatically attracts and is attracted to other bodies) -- and inertial mass is *not* equal to electric charge!

This coincidence is more than curious: it underpins a striking physical phenomenon -- that two bodies, freely falling under gravity from the same starting point, undergo the same acceleration, however disparate their masses ('Galileo's law', as in the legend about dropping balls from the Tower of Pisa). The accelerations are the same because of the equality of inertial and gravitational mass: the more massive body feels a proportionately greater force (thanks to its greater 'gravitational charge') which exactly compensates for its greater inertia.

Einstein accepted this phenomenon of the equal acceleration of freely falling bodies; but he sought a theory of gravity that somehow explained the equality, and thereby its consequence -- the phenomenon.

Second, both Newtonian theory and this approach make spacetime 'absolute' in the sense that it is unaffected by its material contents. That is: Newton's space and time (and also Newtonian spacetime) are utterly uninfluenced by the bodies they contain, no matter how massive the bodies are (and no matter what their other properties are.) Similarly, the above approach takes Minkowski spacetime as absolute. As it is sometimes put: spacetime is a 'fixed canvas' on which physical interactions, of whatever strength and type, get 'painted'.

Yet for both Newtonian theory and this approach, spacetime does in a sense influences its material contents. Namely: its metrical structure (either the two metrics of Newtonian spacetime, or the Minkowski metric) defines a notion of straight line ('geodesic') not only in space, but also *in spacetime*: such spacetime geodesics are the possible trajectories in spacetime ('world-lines') of particles moving freely, subject to no forces at all, not even gravity. So these spacetimes influence their material contents in the sense of determining a standard of undisturbed, unaccelerated motion ('inertial motion').

Einstein wanted to do away with this asymmetry. He could not believe in an entity that was not influenced by material bodies in any way, and that nevertheless influenced them, even in the uniform and minimal way just described. So he sought a theory of gravity in which spacetime would be influenced by the distribution of matter within it.

The third feature follows on from the second. Both Newtonian theory and this approach treat gravity as disturbing bodies' natural inertial motion: it is a force that accelerates them away from the inertial worldlines, that they would follow if they were subject to no forces.

Einstein wanted somehow to treat falling under gravity (subject to no other forces) as itself a 'natural', 'undisturbed' motion. (This vague idea is part of Einstein's so-called 'principle of equivalence'. But the formulation of this principle is subtle and controversial.)

(4: General Relativity)
Einstein's three desiderata for a theory of gravity -- viz. that it somehow explain the equality of inertial and gravitational mass, that it have spacetime be influenced by the matter-distribution, and that it treat free fall under gravity as 'undisturbed' -- were a tall order.

Indeed, his general relativity (obtained after a long struggle in late 1915) fully satisfies only the second and third. (The first is partially satisfied (i) by general relativity's incorporating the principle of equivalence; and (ii) by its entailing, under some assumptions, that the worldlines of freely falling test particles are geodesics. But (i) and (ii) are both complicated.)

General relativity's satisfaction of these two desiderata is a master-stroke. The idea is that matter and spacetime influence each other, and do so locally. The theory postulates that its fundamental equation, the Einstein field equations, which describe the mutual influence, hold at each point in spacetime.

On the one hand, the second desideratum is satisfied because matter influences the metrical structure of the spacetime region in which it is embedded. This varying metrical structure defines straight lines (geodesics) at each point in spacetime; and because the metric varies from point to point, the detailed structure of the set of geodesics through a point also varies.

On the other hand, the third desideratum is satisfied because general relativity takes these geodesics to be the possible trajectories in spacetime ('world-lines') of particles falling freely under gravity, subject to no other forces.

The theory represents how gravity depends on its source (e.g. a greater force from a more massive body) by the fact that particle worldlines vary from point to point, according to the varying structure of the set of geodesics, and so according to the varying nature (e.g. density) of matter.

In this way, gravity is 'geometrized', i.e. rendered as an aspect of spacetime geometry.

For example, the planets orbiting the Sun are falling freely under the Sun's gravity; so their worldlines are geodesics of a spacetime (not spatial!) geometry determined by the Sun.

(5: Non-Euclidean Geometry)
As Einstein realised soon after starting his search for general relativity, making these ideas precise requires the use of generalised geometries that were developed during the nineteenth century, pre-eminently by Riemann (who also discussed their possible application in physics). I turn to a brief sketch of the main ideas involved.

Such geometries were at first called non-Euclidean, in contrast to Euclid's geometry. On first acquaintance, they seem paradoxical: for example, how could measuring the angles of a triangle yield a total greater than 180°? But in the course of the nineteenth century, such geometries were accepted, indeed thoroughly analysed, within pure mathematics. (There was a long tradition of investigations of Euclid's fifth axiom: it seemed less evident than his others, and these investigations showed the apparent consistency of adding rival fifth axioms.)

This was in part due to the widening acceptance of the idea that mathematics is formal: 'line', 'point' and the other geometric terms were endowed with meaning just by their place in an axiom system ('implicit definition'); and the consistency of an axiom system could be shown just by displaying an interpretation that made all the axioms true (‘exhibiting a model’). It turned out that curved surfaces (which had been intensively studied by Gauss early in the century) provided interpretations for various (two-dimensional) non-Euclidean axiom systems.

For example, the surface of a sphere provided an interpretation for a geometry in which the angles of a triangle summed to more than 180°. (Think of the ‘triangle’ on the Earth’s surface formed by the Northern halves of the Greenwich Meridian and the 90° West Line of Longitude, and the portion of the Equator between them; at all three corners, the angle is 90°, so that the total is 270°.) Figure:

[The development of non-Euclidean geometry was the first phase of the 19th-century explosion in geometry: an explosion in detailed content, abstraction, generality, classification and in relations to other branches of mathematics: Gauss, Riemann, Klein, Lie, Hilbert, Poincare ... Most of these developments were marked by an increasing interest -- as elsewhere in mathematics -- in axiomatization.)

Several factors caused this increasing interest in axiomatization (and thereby a clearer contrast between pure maths and applied maths, i.e. theoretical physics): at least three. In algebra, the development of complex numbers and quaternions liberated algebra from a concern with magnitudes as traditionally understood (i.e. real scalars). Paradoxes in naive set theory also prompted axiomatization. And Dedekind’s continuation of the Cauchy-Weierstrass programme for making the calculus rigorous also led to axiomatic systems governing positive integers, and to the set-theoretic construction of such numbers.]

These ideas were soon generalised in three ways.

First, there turned out to be other intuitive features (for example, ‘geodesic deviation’, which concerns the structure of the set of geodesics through a point) that were closely connected with features such as the angles of a triangle not summing to 180°. So all these features were thought of as manifestations of curvature; and non-Euclidean geometries came to be called ‘curved’, while Euclidean geometry was ‘flat’.

Second, since these features could all be defined and studied in a small neighbourhood of a point, one could consider spaces with different amounts of curvature at different points (‘inhomogeneous spaces’; e.g. the surface of a rugby ball).

Third, there was Riemann’s distinctive contribution: he showed that one could define these features on spaces of higher dimension (3 or more). These spaces can be mathematical, not physical; in effect, they can be sets of n-tuples of real numbers, with n equal to 3 or more. As such, these spaces can be defined without recourse to any higher dimensional “embedding” space. (So although we picture, for example, a two-sphere (S^2) as the surface of a two-ball in Euclidean three-space, the Euclidean three-space - and *a fortiori* the ball - is actually an unnecessary prop, useful only to facilitate the imagination.)

Curvature plays a role in these higher-dimensional spaces: one can say that space is curved if e.g. the angles of a triangle do not sum to 180°. (Again, such spaces could be inhomogeneous, i.e. could have variable curvature.)

(6: Non-Euclidean Geometry in Physics)
Returning to general relativity, the generalised geometries just sketched have two main applications: to spacetime, and to (physical) space. Given our previous discussion, the application to spacetime is clear enough.

Spacetime is a 4-dimensional space of the kind envisaged by Riemann: the metrical structure, and so the structure of the geodesics, and so the curvature, varies from point to point, constrained by the matter-distribution in accordance with the Einstein field equations.

(Beware terminology: spacetime is not actually 'a Riemannian space', as that term is formally defined; rather, it (its metric) is semi-Riemannian (aka Lorentzian).)

The application to physical space is also clear enough: 'physical space' now means 'spatial slices of spacetime', and if spacetime's metrical structure is influenced by the matter-distribution, it is very likely that the geometry of these spatial slices will vary from point to point. So in general, it will be non-Euclidean.

Here we should add another historical point, which brings out, once more, Einstein's stunning intellectual imagination. Although by 1905 non-Euclidean geometries were entirely accepted within pure mathematics, it remained controversial that the geometry of physical space could be non-Euclidean.

Indeed, Poincaré had argued that it could not be, in the sense that any physical theory that attributed a non-Euclidean geometry for space could be replaced by an empirically equivalent theory retaining a Euclidean geometry. This is Poincaré's conventionalist philosophy of geometry: it had of course immense influence on the positivists, especially Reichenbach, and on Grünbaum. (Poincaré also suggested that however physics developed, such a replacement would be made, because of the simplicity and familiarity of Euclidean geometry. In this the development of general relativity has apparently proved him wrong; for a sympathetic assessment by Einstein himself, see Einstein (1922).)

(7; Philosophical and Historical notes: so much the worse for Kant?)
Kant famously held that Euclidean geometry was a priori knowledge (also synthetic). So the advent of general relativity, forcing non-Euclidean geometries for space, has made Kant an easy target for disdain.

That is unfair on three counts, of ascending importance:

(i) Kant can perhaps be defended as making a true claim about how we must conceive/imagine/visualize space -- perhaps even an a priori one.

(ii) Kant can be defended as engaging in admirable detail with the science of his day; (recent work by Friedman).

(iii) Kant was not alone! After all, the image of mathematics as providing a body of timeless and absolutely certain truths goes back at least to Plato. The Greeks' axiomatically organized geometry, as handed down to us by Euclid, was the paradigm case of such a body of knowledge.

Three other points reinforce (iii).

(a) There was little impetus to ask how such apparently necessary truths could be true of the prevalently contingent world of material objects in space.The reason is partly that most modern philosophers after Descartes were comparatively uninterested in issues about modality, and even about logic [the great exception is Leibniz]. Partly the question was finessed into an epistemic one, about how such necessary truths could be known by inhabitants of such a world. In the answer to this, God of course played a prevalent and important, though variable!, role.

(b) The above points apply, not just to geometry, but also to arithmetic and algebra. We tend to forget that the idea of geometry as secondary to arithmetic and algebra is recent: it is not just post-Descartes -- it arises only from the late-nineteenth century constructions of all mathematics from the integers (and ultimately from pure set-theory). Until then, a real number was often taken to be a ratio (in effect: what we would now call an ordered pair) of spatial intervals; e.g. Newton's *Universal Arithmetic*. (This adds a historical irony to Hartry Field's position!)

(c) After the rise of the mechanical philosophy, the success of Galileo's manifesto 'Nature is a book, written in the language of mathematics', the above points apply also to the 'laws of physics'! Most 17th-century mechanical philosophers apparently believed (with what now seems undue optimism!) that they would soon secure absolutely certain knowledge of timeless laws -- whose modal status was not firmly distinguished from that of the mathematics they already knew. (Point (c) also applies to poor Kant: Euclidean geometry is just the best-known of several principles that he cherished as indispensable to experience, but that were apparently discarded by relativity and quantum theory: e.g. mass conservation, and determinism.)

(8: Summary and Cosmology!)
To sum up: I have presented four ideas: from special relativity comes the idea of spacetime, and the abandonment of absolute simultaneity; from general relativity comes the idea that spacetime is dynamical, and that gravity should be 'geometrized' (Eddington 1922). I sketched how these last two ideas exploit non-Euclidean geometries.

These four ideas all fall under our topic, the philosophy of space and time. But any 'hymn to Einstein' would be incomplete, without remarking on the development of cosmology as a genuine science. While the growth of observational astronomy has of course been crucial (above all, in the discovery of the expansion of the universe, and of the microwave background radiation), general relativity has also played a crucial theoretical role. For by making spacetime dynamical, it allowed one to envisage various precise global structures for spacetime, which could then be compared with observations. Einstein's general relativity, dating from 1915, still remains our best theory, in application both to specific astronomical phenomena and to the universe as a whole.

(AII) Absolute vs. Relational

Recall: The traditional philosophical folklore is that Newton was an absolutist, and Leibniz a relationist; that in the course of time, the success of Newton's science overshadowed Leibniz's arguments; but that the advent of relativity theory vindicated Leibniz's relationism.

Let us review the rebuttal (nowadays, quite well-known in the literature), of this folklore, by first

1. Distinguishing (some of the several!) different senses of 'absolute'. The first sense is common and very significant, but not to the point in this rebuttal of the folklore:-
(0) independent of choice of coordinate-system, aka: observer-independent; jargon: represented by a `geometric object' (scalar, vector, tensor---considered as coordinate-independent object, which has components in various coordinate systems). Then there are:-

(1) the existence of absolute space, absolute rest; i.e. of a distinguished congruence of timelike curves through spacetime.

(2) absolute = not affected by the matter distribution, not dynamical. (Often made precise in terms of 'varying from one possible world, model of the theory, to another'.)

(3) absolute = not determined by (not supervenient upon) the spatial and temporal relations of bodies (matter). (cf. Newton's bucket experiment or globe experiment, in the last two pages of his Scholium to Definition 8: Alexander ed. pp. 157-160, and the commentray on pp. xxxvii.) (Mach hoped for a theory without such effects.)

(4) the existence of spacetime (or spacetime) points as objects on a par with the rest of one's ontology (so called 'substantivalism').

The popularity of this claim reflects the rise of scientific realism from the mid-1960's onwards. For scientific realism holds that one is committed to believing in the existence of those entities that are ineliminably referred to or quantified over by one's best scientific theories. And our best physical theories are presented as quantifying over spacetime points -- with never a hint of how to eliminate such quantification.

2. In short, the situation is: the advent of relativity has vindicated relationism in the sense of the denials of (1), (2).

But absolutism of kind (3) holds for Newtonian mechanics, special relativity and general relativity. And (4) is, though controversial, as defensible for relativity theories as it was before; (some say, more so). Recently the controversy has centred around Earman & Norton's resuscitation (1987) of Einstein's 'hole argument'.

I shall spell out the denials of (1), (2). These give in effect two respects in which modern developments have vindicated relationism.

Against (1):-- The first vindication of relationism arises from the use of modern differential geometry to present spacetime theories. In any theory, classical or relativistic, the treatment of motion requires that spacetime have a four-dimensional *affine connection*. Without a connection, the distinction between unaccelerated and accelerated motion (between a straight line, and a curved line, in four-dimensions) cannot be made independent of coordinate system,as it needs to be.

Newton wasjustified in positing an absolute rest in the sense that in his own time (and up through 1900), this was the only known way of positing a connection.

But it is in fact not the only way. Modern geometry shows how a spacetime, classical or relativistic, can have a connection, and the associated concept of absolute acceleration, without having an absolute rest.

Thus relationism considered as the denial of absolute rest is vindicated, even for classical spacetimes.

Against (2):-- The third vindication depends on general relativity. In general relativity, spacetime structure is responsive to the distribution of matter; so that spacetime is not a fixed container for physical events, unaffected by matter but itself controlling matter's motion through its metric and connection.

Note however that these vindications of relationism do not imply that spacetime structure, or the magnitudes needed to treat motion, are determined by the spatiotemporal relations of bodies, as Mach apparently hoped. This version of relationism (denial of (2)) fails in general relativity, no less than in the classical case. For Newton's thought-experiment with the bucket or globes -- showing that the spatiotemporal relations of bodies can be the same in two cases, while the bodies' worldlines are geodesics in one case but not the other -- is valid in general relativity.